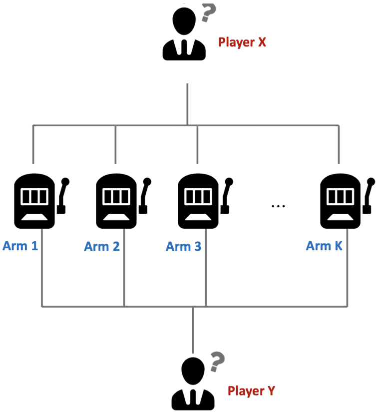
# The Pareto Frontier of Instance-Dependent Guarantees in Multi-Player Multi-Armed Bandits with no Communication

Mark Sellke (Stanford)

With Allen Liu (MIT)

# Multi-Player (Cooperative) Bandits

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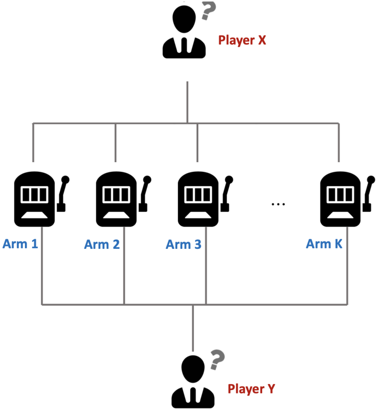


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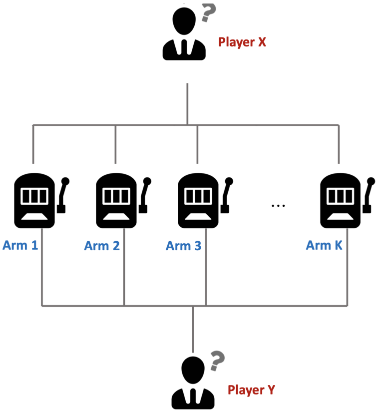


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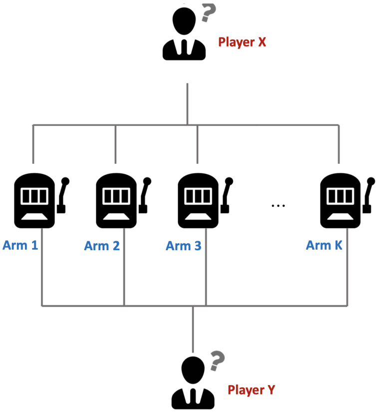


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Proposed for wireless radio – learn good signal frequencies while avoiding interference. [Lai-Jiang-Poor 08, Liu-Zhao 10, Anandkumar-Michael-Tang-Swami 11].

# Partial Formulation

Fix $\mathbf{p} = (p_1, p_2, \ldots, p_K) \in [0, 1]^K$. Generate $TKm$ independent Bernoulli reward variables $\mathrm{rew}_t^X(i)$ for $(t, i, X) \in [T] \times [K] \times [m]$:

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Key quantities of interest: minimax and instance-dependent regret

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More specification needed! What information is observed about collisions?

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4. **Adversarial**: observe a reward chosen by an adaptive adversary.

# Regret for these Models

Strongly detectable: regret $\tilde{O}(\sqrt{T})$, even for non-stochastic. **Implicit communication**.
($\tilde{O}(\cdot)$ hides $\mathrm{poly}(K, \log T)$ factors.) [Lugosi-Mehrabian 18, Bubeck-Li-Peres-S. 19]

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**Theorem (Bubeck-Budzinski 20, Bubeck-Budzinski-S. 21)**

*There exists an efficient, collision-free strategy with $\tilde{O}(\sqrt{T})$ regret. Precisely,*

$$R_T = O\left(mK^{11/2}\sqrt{T \log T}\right),$$

$$\mathbb{P}(\textit{there is ever a collision}) = O(T^{-2}).$$

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*The Pareto optimal regret guarantees with no communication are:*

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Corollary: undetectable and adversarial models behave the same (up to $\mathrm{poly}(K, \log T)$).

Corollary: if $\Delta \ll \Delta'$, no algorithm achieves $R_{T,\Delta} \le \tilde{O}(1/\Delta)$ **and** $R_{T,\Delta'} \le \tilde{O}(1/\Delta')$.

# Geometric Viewpoint with 2 players and 3 actions

For illustration, work in the plane $P = \{p_1 + p_2 + p_3 = \text{constant}\}$ under full feedback.

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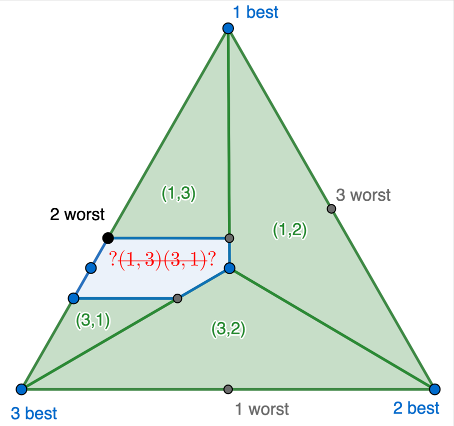
$$(i_t^X, i_t^Y) : P \to \{1, 2, 3\}^2.$$



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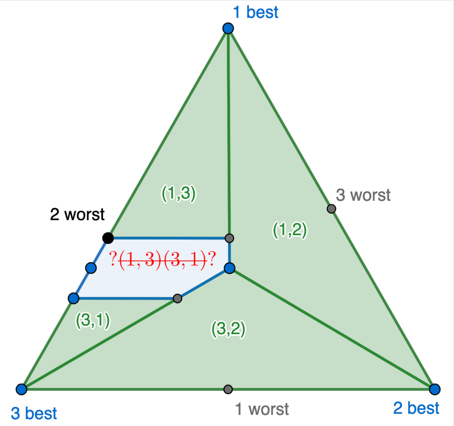


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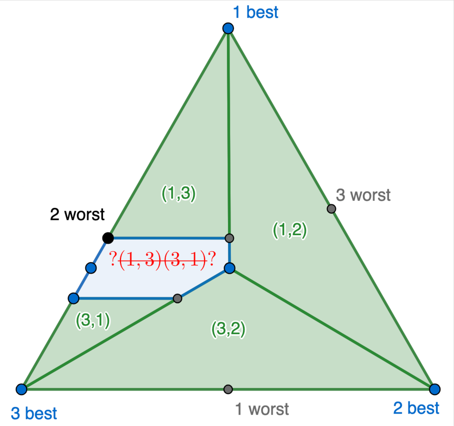


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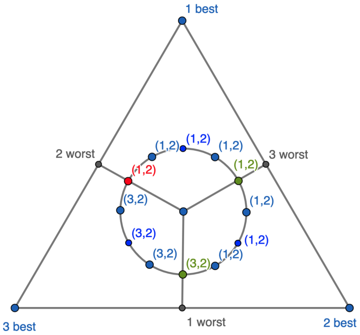
Difficulty: cannot always play the top 2 arms without colliding for some $\mathbf{p}$.

# One-Step Regret Lower Bound with 2 players and 3 actions

How to turn this into a lower bound? Consider $\sqrt{T}$ points equally spaced on a constant-size circle, labelled according to the time $T$ strategy.



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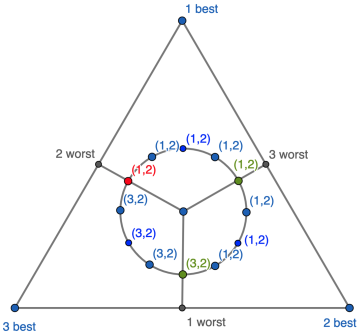


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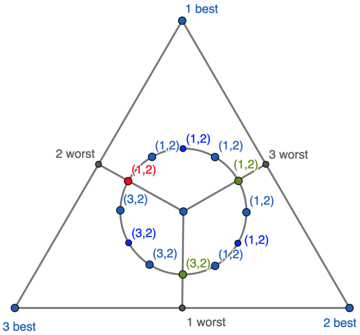


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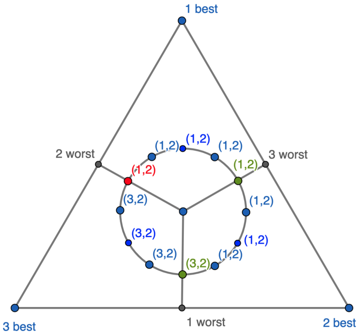


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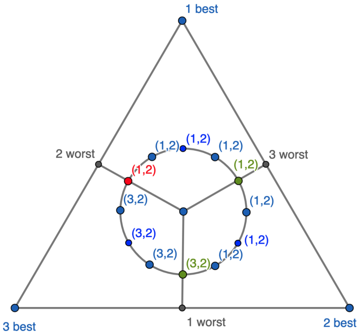


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By dyadic pigeonhole, there **exists** a gap $\Delta_J$ with $\tilde{\Omega}(T)$ **FAIL**s across $1 \leq t \leq T$.

There are $\approx \Delta_J \sqrt{T}$ points on the circle with gap $\approx \Delta_J$ to absorb the **FAIL**s. Hence

$$R_{T,\Delta_J} \gtrsim \frac{T}{\Delta_J \sqrt{T}} = \frac{\sqrt{T}}{\Delta_J}.$$

# General Lower Bound: Set $T_J = \Delta_J^{-2}$ and Iterate

**General Pareto Optimal $R_{T,\Delta}$.**

$\mathbf{R_{T,\Delta_J}} \gtrsim \frac{\sqrt{T}}{\Delta_J}$ for **some** $T^{-1/2} \le \boldsymbol{\Delta}_J \le 1.$

Iterate: $\mathbf{R_{T,\Delta_{J-1}}} \gtrsim \frac{1}{\Delta_{J-1}\Delta_J}$ for **some** $\boldsymbol{\Delta}_J \le \boldsymbol{\Delta}_{J-1} \le 1.$

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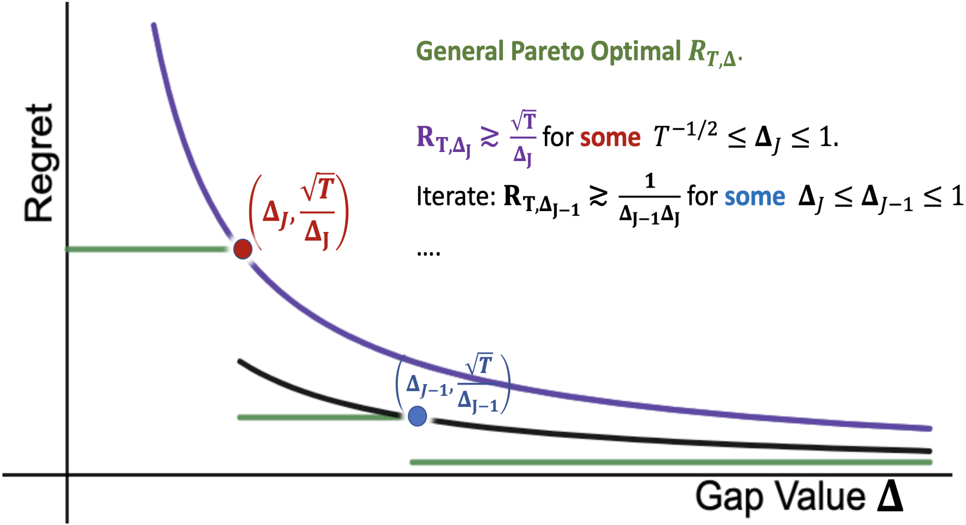
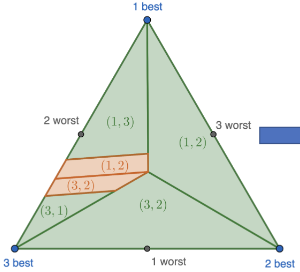
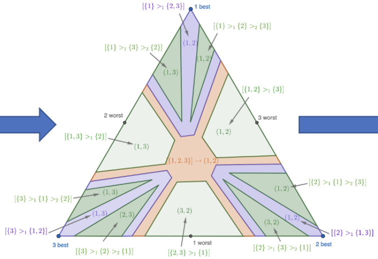


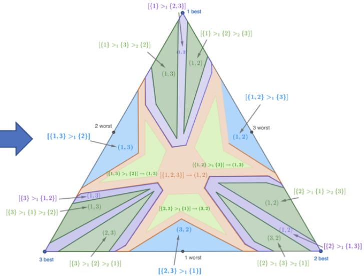
# Collision-Free Algorithms At a Glance




[Bubeck-Budzinski 20]



[Bubeck-Budzinski-**S** 21]



[Liu-**S** 22]

- Previously: in multi-player stochastic bandits, $\tilde{O}(\sqrt{T})$ regret is possible with no collisions. Implicit communication enables $\tilde{O}(1/\Delta)$.
- This paper: without communication, Pareto optima include $\tilde{O}(\sqrt{T})$ and $\tilde{O}(1/\Delta^2)$. In particular, $\tilde{O}(1/\Delta)$ is only possible at a single scale $\Delta$.