STAT 291 **Spring 2024**

Suggested Project List

Instructions: choose a project topic and aim to let us know by March 1, or at latest by the start of Spring break. You can work with up to two partners. The topics below are not meant to be exhaustive, and you are welcome to take the ideas in your own direction instead of the ones suggested.

Project 1: Fast Mixing of Glauber Dynamics

It will be explained in class that Langevin dynamics for spherical spin glasses mix rapidly at sufficiently high temperature. In particular, this yields computationally efficient sampling from high temperature Gibbs measures. Recent exciting progress has shown the same for sampling from Ising spin glasses, which is much more challenging.

For the SK model, this was shown in [BB19, EKZ21, CE22]. The later (rather technical) papers [ABXY22, AJK+23] can handle mixed p-spin models. Another diffusion-based sampling method was given in [EAMS22].

One possible project based on these works would cover [BB19, EKZ21]. Another possibility is to focus on one or both of [EAM22, Mon23] (which explain diffusion sampling in general) and then explain its application to the SK model in [EAMS22].

Project 2: Superconcentration

In class, we applied Lipschitz concentration to show concentration of spin glass free energies $F_N(\beta)$. In fact the variance is even smaller than we showed, as proved by Chatterjee using *superconcentration*. The book treatment [Cha14] covers a lot of material and could be used in several ways for a survey project.

For students who know something about representations of Lie groups, there is a potential research project here as well. Come talk to me if you are interested!

Project 3: Random Perceptron

In Homework 1, you may have solved the random spherical perceptron when $\kappa = 0$. There is an exact solution for any $\kappa \geq 0$ obtained by [Sto13a], who also obtained an improved upper bound in [Sto13b]. Stojnic introduced the convex Gordon minimax inequality which has seen many applications in high-dimensional statistics and a potential project could investigate this method.

Another interesting direction is the symmetric binary perceptron. Frozen 1-RSB was conjectured in [APZ19] and proved in [PX21]; it asserts that nearly all solutions are strongly isolated, at linear Hamming distance from all others. [KR98, ALS22b] gave efficient algorithms to construct solutions. [GKPX22, GKPX23] gave algorithmic hardness results using the overlap gap property (which we will cover after Spring break). Several combinations of these papers would make interesting projects.

Project 4: Small Subgraph Conditioning

In the absence of concentration inequalities, another way to improve the second moment method is known as "small subgraph conditioning". The idea is that if the ratio

$\mathbb{E}[Z^2]/\mathbb{E}[Z]^2$ converges to a constant $C > 1$, one might be able to show the constant is "fully explained" by simple observables, so that the ratio tends to 1 after **conditioning** on the observables. For sparse random graphs, these observables are usually counts of small subgraphs. This method was introduced in [Wor81] to show the high-probability existence of Hamiltonian cycles in sparse random graphs, and is explained in [Wor99, Chapter 4]. Recent applications include [MNS15, ALS22a]. A survey project might explain the general method and some applications.

Project 5: Algorithms for Random k-SAT

Many algorithms for finding solutions to random k-SAT have been developed, including heuristics based on belief/survey propagation, rigorous algorithms [COFF$^+$09, CO10], and algorithms for all instances with uniformly bounded degrees [Mos09, Moi19]. Corresponding algorithmic hardness results using the overlap gap property (which we will cover after Spring break) are also known [COHH17, GS17, BH21]. Explaining some of these algorithms or hardness results would make a good survey project.

Project 6: Adaptive Interpolation Method

In class, we will use the interpolation method to obtain upper bounds for free energies. Recently, [BM19] and follow-up work showed how to obtain matching interpolation lower bounds for the important setting of posteriors arising in Bayesian inference. A crucial ingredient is the Nishimori identity, which was used in Homework 1 problem 4(e). Explaining this method would make a nice project.

Project 7: Strong Topological Trivialization

In class, we (will) discuss the topologically trivial phase of spherical spin glasses, where the Kac–Rice formula shows the number of critical points is $e^{o(N)}$. [HS23] shows how to obtain stronger landscape properties that ensure fast convergence of optimization algorithms. The paper considers more general "multi-species" spin glasses, and explaining in detail the specialization to ordinary mixed p-spin models would make a good project. This would require knowing/learning more advanced random matrix theory than was coverd in lecture. Research projects are likely also possible.

Project 8: Langevin Dynamics

In class we touched on several aspects of Langevin dynamics. For spherical spin glasses, there is a rich literature including [ADG01, BDG06, DGM07, CCM21, Sel23]. An important related work on non-convex optimization is [JNG$^+$21], while physicists calculate using dynamical mean field theory. Surveying some of these ideas would make a nice project. Research projects are also possible here, such as extensions to Riemannian manifolds and connections to the Kac–Rice formula (the latter would be closely related to the previous project).

References

[ABXY22] Arka Adhikari, Christian Brennecke, Changji Xu, and Horng-Tzer Yau. Spectral gap estimates for mixed p-spin models at high temperature. *arXiv preprint arXiv:2208.07844*, 2022.

[ADG01] G Ben Arous, Amir Dembo, and Alice Guionnet. Aging of spherical spin glasses. *Probability theory and related fields*, 120:1–67, 2001.

[AJK⁺23] Nima Anari, Vishesh Jain, Frederic Koehler, Huy Tuan Pham, and Thuy-Duong Vuong. Universality of spectral independence with applications to fast mixing in spin glasses. *arXiv preprint arXiv:2307.10466*, 2023.

[ALS22a] Emmanuel Abbe, Shuangning Li, and Allan Sly. Proof of the contiguity conjecture and lognormal limit for the symmetric perceptron. In *2021 IEEE 62nd Annual Symposium on Foundations of Computer Science (FOCS)*, pages 327–338. IEEE, 2022.

[ALS22b] Emmanuel Abbe, Shuangping Li, and Allan Sly. Binary perceptron: efficient algorithms can find solutions in a rare well-connected cluster. In *Proceedings of the 54th Annual ACM SIGACT Symposium on Theory of Computing*, pages 860–873, 2022.

[APZ19] Benjamin Aubin, Will Perkins, and Lenka Zdeborova. Storage capacity in symmetric binary perceptrons. *Journal of Physics A: Mathematical and Theoretical*, 52(29):294003, 2019.

[BB19] Roland Bauerschmidt and Thierry Bodineau. A very simple proof of the LSI for high temperature spin systems. *Journal of Functional Analysis*, 276(8):2582–2588, 2019.

[BDG06] Gérard Ben Arous, Amir Dembo, and Alice Guionnet. Cugliandolo-Kurchan equations for dynamics of spin-glasses. *Probability theory and related fields*, 136(4):619–660, 2006.

[BH21] Guy Bresler and Brice Huang. The Algorithmic Phase Transition of Random k-SAT for Low Degree Polynomials. In *2021 IEEE 62nd Annual Symposium on Foundations of Computer Science (FOCS)*, pages 298–309, 2021.

[BM19] Jean Barbier and Nicolas Macris. The adaptive interpolation method: a simple scheme to prove replica formulas in bayesian inference. *Probability theory and related fields*, 174(3):1133–1185, 2019.

[CCM21] Michael Celentano, Chen Cheng, and Andrea Montanari. The high-dimensional asymptotics of first order methods with random data. *arXiv preprint arXiv:2112.07572*, 2021.

[CE22] Yuansi Chen and Ronen Eldan. Localization schemes: A framework for proving mixing bounds for markov chains. In *2022 IEEE 63rd Annual Symposium on Foundations of Computer Science (FOCS)*, pages 110–122. IEEE, 2022.

[Cha14] Sourav Chatterjee. *Superconcentration and related topics*, volume 15. Springer, 2014.

[CO10] Amin Coja-Oghlan. A better algorithm for random k-SAT. *SIAM Journal on Computing*, 39(7):2823–2864, 2010.

[COFF⁺09] Amin Coja-Oghlan, Uriel Feige, Alan Frieze, Michael Krivelevich, and Dan Vilenchik. On smoothed k-cnf formulas and the walksat algorithm. In *Proceedings of the Twentieth Annual ACM-SIAM Symposium on Discrete Algorithms*, pages 451–460. SIAM, 2009.

[COHH17] Amin Coja-Oghlan, Amir Haqshenas, and Samuel Hetterich. Walksat stalls well below satisfiability. *SIAM Journal on Discrete Mathematics*, 31(2):1160–1173, 2017.

[DGM07] Amir Dembo, Alice Guionnet, and Christian Mazza. Limiting dynamics for spherical models of spin glasses at high temperature. *Journal of Statistical Physics*, 126:781–815, 2007.

[EAM22] Ahmed El Alaoui and Andrea Montanari. An information-theoretic view of stochastic localization. *IEEE Transactions on Information Theory*, 68(11):7423–7426, 2022.

[EAMS22] Ahmed El Alaoui, Andrea Montanari, and Mark Sellke. Sampling from the sherrington-kirkpatrick gibbs measure via algorithmic stochastic localization. In *2022 IEEE 63rd Annual Symposium on Foundations of Computer Science (FOCS)*, pages 323–334. IEEE, 2022.

[EKZ21] Ronen Eldan, Frederic Koehler, and Ofer Zeitouni. A spectral condition for spectral gap: fast mixing in high-temperature Ising models. *Probability Theory and Related Fields*, pages 1–17, 2021.

[GKPX22] David Gamarnik, Eren C Kızıldağ, Will Perkins, and Changji Xu. Algorithms and barriers in the symmetric binary perceptron model. In *2022 IEEE 63rd Annual Symposium on Foundations of Computer Science (FOCS)*, pages 576–587. IEEE, 2022.

[GKPX23] David Gamarnik, Eren C Kizildağ, Will Perkins, and Changji Xu. Geometric barriers for stable and online algorithms for discrepancy minimization. In *The Thirty Sixth Annual Conference on Learning Theory*, pages 3231–3263. PMLR, 2023.

[GS17] David Gamarnik and Madhu Sudan. Performance of sequential local algorithms for the random NAE-K-sat problem. *SIAM Journal on Computing*, 46(2):590–619, 2017.

[HS23] Brice Huang and Mark Sellke. Strong topological trivialization of multi-species spherical spin glasses. *arXiv preprint arXiv:2308.09677*, 2023.

[JNG+21] Chi Jin, Praneeth Netrapalli, Rong Ge, Sham M Kakade, and Michael I Jordan. On nonconvex optimization for machine learning: Gradients, stochasticity, and saddle points. *Journal of the ACM (JACM)*, 68(2):1–29, 2021.

[KR98] Jeong Han Kim and James R Roche. Covering cubes by random half cubes, with applications to binary neural networks. *Journal of Computer and System Sciences*, 56(2):223–252, 1998.

[MNS15] Elchanan Mossel, Joe Neeman, and Allan Sly. Reconstruction and estimation in the planted partition model. *Probability Theory and Related Fields*, 162:431–461, 2015.

[Moi19] Ankur Moitra. Approximate counting, the lovász local lemma, and inference in graphical models. *Journal of the ACM (JACM)*, 66(2):1–25, 2019.

[Mon23] Andrea Montanari. Sampling, diffusions, and stochastic localization. *arXiv preprint arXiv:2305.10690*, 2023.

[Mos09] Robin A Moser. A constructive proof of the lovász local lemma. In *Proceedings of the forty-first annual ACM symposium on Theory of computing*, pages 343–350, 2009.

[PX21] Will Perkins and Changji Xu. Frozen 1-rsb structure of the symmetric ising perceptron. In *Proceedings of the 53rd Annual ACM SIGACT Symposium on Theory of Computing*, pages 1579–1588, 2021.

[Sel23] Mark Sellke. The Threshold Energy of Low Temperature Langevin Dynamics for Pure Spherical Spin Glasses. *arXiv preprint arXiv:2305.07956*, 2023.

[Sto13a] Mihailo Stojnic. Another look at the gardner problem. *arXiv preprint arXiv:1306.3979*, 2013.

[Sto13b] Mihailo Stojnic. Negative spherical perceptron. *arXiv preprint arXiv:1306.3980*, 2013.

[Wor81] Nicholas C Wormald. The asymptotic distribution of short cycles in random regular graphs. *Journal of Combinatorial Theory, Series B*, 31(2):168–182, 1981.

[Wor99] Nicholas C Wormald. Models of random regular graphs. *London Mathematical Society Lecture Note Series*, pages 239–298, 1999.